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MEMORANDUM

MODELS OF ACCESSION AND RETENTION

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TABLE OF CONTENTS

MODELS OF ACCESSION AND RETENTION

INTRODUCTION

In this paper, we outline two simple models that integrate accession and first-term retention policies. The first model describes the relation between optimal accession and first-term reenlistment bonus policies when marginal recruiting costs are constant. It is appropriate for the analyst who is concerned with the bonus policy for a single, relatively small rating or for the analyst dealing with a group whose supply is demand determined. Using this model, we find that optimal reenlistment bonus levels will be the highest in ratings which have high first-term attrition, high training costs, and which would have low reenlistment rates in the absence of bonuses. We also find that optimal bonus levels rise as military wages fall relative to civilian wages.

The second model is similar to the first, but takes account of the fact that marginal recruiting costs rise as more recruits are obtained. Because of this, we find that a larger Navy implies higher optimal reenlistment bonus levels even in the long run.

MODEL I: CONSTANT RECRUITING COSTS

In this model, the Navy seeks to minimize the costs of meeting fixed LOS-5 requirements. Cost = $\gamma X + RMX\delta$, where γ is the cost of "growing" an individual eligible for reenlistment, X is the number of eligibles, R=R(M) is the reenlistment rate, M is the value of a unit level bonus multiple, and δ, the discount factor, is equal to e^{-rt}.

In this simple model, all eligibles follow a path which includes formal A-school training immediately after initial recruit training. Wages after A-school are assumed to equal to the value of the individual marginal product. γ, the net cost to the Navy of growing an eligible, is thus equal to $\frac{\Theta_1+\Theta_2}{P_1} + \frac{\Theta_3}{P_2}$. Θ_1 is the (constant) cost of recruiting an individual, Θ_2 is the cost of initial recruit training for an individual, and Θ_3 is the cost of class A-school training; P_1 is the probability that a recruit will survive from enlistment to be eligible for reenlistment at LOS-4, and P_2 is the probability that an individual starting A-school will survive to be eligible for reenlistment. Because of attrition during initial recruit training, it is worth more to the Navy to reduce the cost of recruiting by $1/recruit than it is to reduce A-school costs by $1/trainee.

The Navy's objective in this model is to minimize costs subject to the constraint that LOS-5 requirements (F) are met.

Objective Function: $C = \gamma X + RMX\delta$

Constraint: $F = R(M)X$

The Lagrangian is: $L = \gamma X + RMX\delta + \lambda[F-RX]$. Letting R_M signify $\frac{\partial R}{\partial M}$ we obtain the following first order conditions:

(1) $\frac{\partial L}{\partial M} = 0 = R_M MX\delta + RX\delta - \lambda R_M X$,

(2) $\frac{\partial L}{\partial X} = 0 = \gamma + RM\delta - \lambda R$, and

(3) $\frac{\partial L}{\partial \lambda} = 0 = F - RX$.

From (1) and (2) above we obtain the equality:

(4) $\gamma = R^2\delta/R_M$.

Differentiating, we find that $d\lambda = (2\delta R - \delta R^2/R_{MM})dM$. Thus, if $R_{MM} < 0$, $\frac{dM}{d\gamma} > 0$. So long as the impact of bonuses on the reenlistment rate declines as bonus levels rise, a higher cost per eligible will be associated with a higher optimal bonus level.

At CNA we have estimated R(M) using the logistic functional form: $\frac{R}{1-R} = e^{\alpha+\beta(RMC^*+M^*)}$. RMC* is the annualized value of regular military compensation less civilian pay over four years and M* is the annualized

value of the unit bonus multiple over a presumed four year reenlistment. $\frac{M}{\nu} = M^*$ where $\nu = \int_0^4 e^{-rt}$. R_M is in this case equal to $\beta R(1-R)/\nu$. This allows us to use (4) to solve explicitly for the optimal value of the reenlistment bonus:

$$\frac{\gamma}{R} = \frac{R\delta}{\frac{\beta}{\upsilon} R(1-R)} \text{ or } \frac{\gamma\beta}{\upsilon\delta} = \frac{R}{1-R} = e^{\alpha+\beta(RMC^*+M^*)} .$$

Thus,

$$(5) \quad M = \frac{\upsilon}{\beta} [\ln\gamma - \ln\upsilon - \ln\delta + \ln\beta - \alpha - \beta RMC^*].$$

Differentiating (5), we find that $dM = \frac{\upsilon}{\beta} \frac{1}{\gamma} d\gamma$. In this case if β, the bonus response, is positive so too is the relation between γ and M. Further, $\frac{d^2M}{d\gamma^2} = -\frac{\nu}{\beta}\gamma^{-2} < 0$; while increases in the cost/eligible mean higher optimal bonus levels, the effect diminishes as the cost/eligible rises.

$\frac{dM}{d\alpha}$, on the other hand, is equal to $-\frac{\nu}{\beta}$. With $\beta > 0$, an increase in the reenlistment rate due to factors other than pay or bonuses is associated with a fall in the optimal bonus multiple. Increases in military pay relative to civilian pay will also lower the optimal bonus level: $dM = -\upsilon dRMC^*$.

The effect of an increase in β (a measure of the responsiveness of reenlistments to a change in the bonus multiple) on the optimal bonus

level is indeterminate. $\frac{\nu}{\beta^2}(\ln\gamma + \ln\upsilon + \ln\delta - \ln\beta + 1 + \alpha)d\beta = dM$. We cannot predict whether rating groups which are the most responsive to bonuses will have the highest bonus multiples.

MODEL II: INCREASING MARGINAL RECRUITING COSTS

This model differs from the first in that, consistent with the findings of previous research, we incorporate increasing marginal recruiting costs. We let $Q(X)$ be the cost of recruiting $\frac{X}{P}$ recruits, where P is the (constant) probability of a recruit surviving to become eligible for reenlistment. $Q_X > 0$ and $Q_{XX} > 0$.

The problem is now to minimize costs where Cost = $Q(X) + \gamma X + RMX\delta$. The variables R, M, X, and δ are defined as before, but γ is now equal to the training costs incurred in producing an individual eligible for reenlistments.

The constraint, $F = RX$, is unchanged from the first model. Setting up the Lagrangian and taking the first partials we have:

$$L = Q(X) + \gamma X + RMX\delta + \lambda[F-RX] \quad \text{with}$$

$$(1) \quad \frac{\partial L}{\partial M} = 0 = MR_M X\delta + RX\delta - \lambda R_M X ,$$

$$(2) \quad \frac{\partial L}{\partial X} = 0 = Q_X + \gamma + RM\delta - \lambda R , \text{ and}$$

(3) $\frac{\partial L}{\partial X} = 0 = F - RX.$

From (1) and (2) we obtain the equality

(4) $\frac{Q_X+\gamma}{R} = \frac{R\delta}{R_M}$.

If we adopt the logistic formulation for R(M), so that

$R = \frac{1}{1+e^{-[\alpha+\beta(RMC*+M*)]}}$, $R_M = \frac{\beta}{\upsilon} R(1-R)$. Together with (4), this yields:

$$\frac{(Q_X + \gamma)\beta}{\delta\upsilon} = \frac{R}{1-R} .$$

Taking the natural log of this expression, we have:

(5) $\ln(Q_X+\gamma) + \ln\beta - \ln(\delta\ \upsilon) = \alpha + \beta RMC* + \frac{\beta}{\upsilon} M.$

Differentiating this equilibrium condition with respect to M, and keeping in mind that $Q_X = Q_X(X) = Q_X\ (\frac{F}{R(M)})$, we obtain the following:

$$\frac{1}{(Q_X+\gamma)}\ [Q_{XX}(\frac{R-F\frac{\partial R}{\partial M}\frac{\partial M}{\partial F}}{R^2}) \approx = \frac{\beta}{\upsilon}\frac{dM}{dF} .$$

Substituting $\frac{\beta}{\upsilon} R(1-R)$ for $\frac{\partial R}{\partial M}$ and simplifying, we find:

(6) $\frac{\nu}{\beta R(Q_X+\gamma)} = \left(Q_{XX}{}^{-1} + \frac{F(1-R)}{R(Q_X+\gamma)}\right)\frac{dM}{dF}$.

γ, υ, R, (1-R), and F are all greater than 0. With a positive and increasing marginal recruiting cost ($Q_X > 0$ and $Q_{XX} > 0$) we conclude that $\frac{dM}{dF} > 0$. A move toward a larger force will tend to increase the optimal bonus multiple. This result is not seen in the first model, where marginal recruiting costs are assumed to be constant.

The second model is essentially an aggregate and simplified version of the model used in the Navy Comprehensive Compensation and Supply Study (NACCS). The empirical relationships between optimal bonus levels and other variables (including recruiting costs, attrition, training costs, and requirements) found in the NACCS Study are consistent with the theoretical relationships established in this paper.